Advanced Algorithms

December 2, 2025

Welcome to the Final Class

Today:

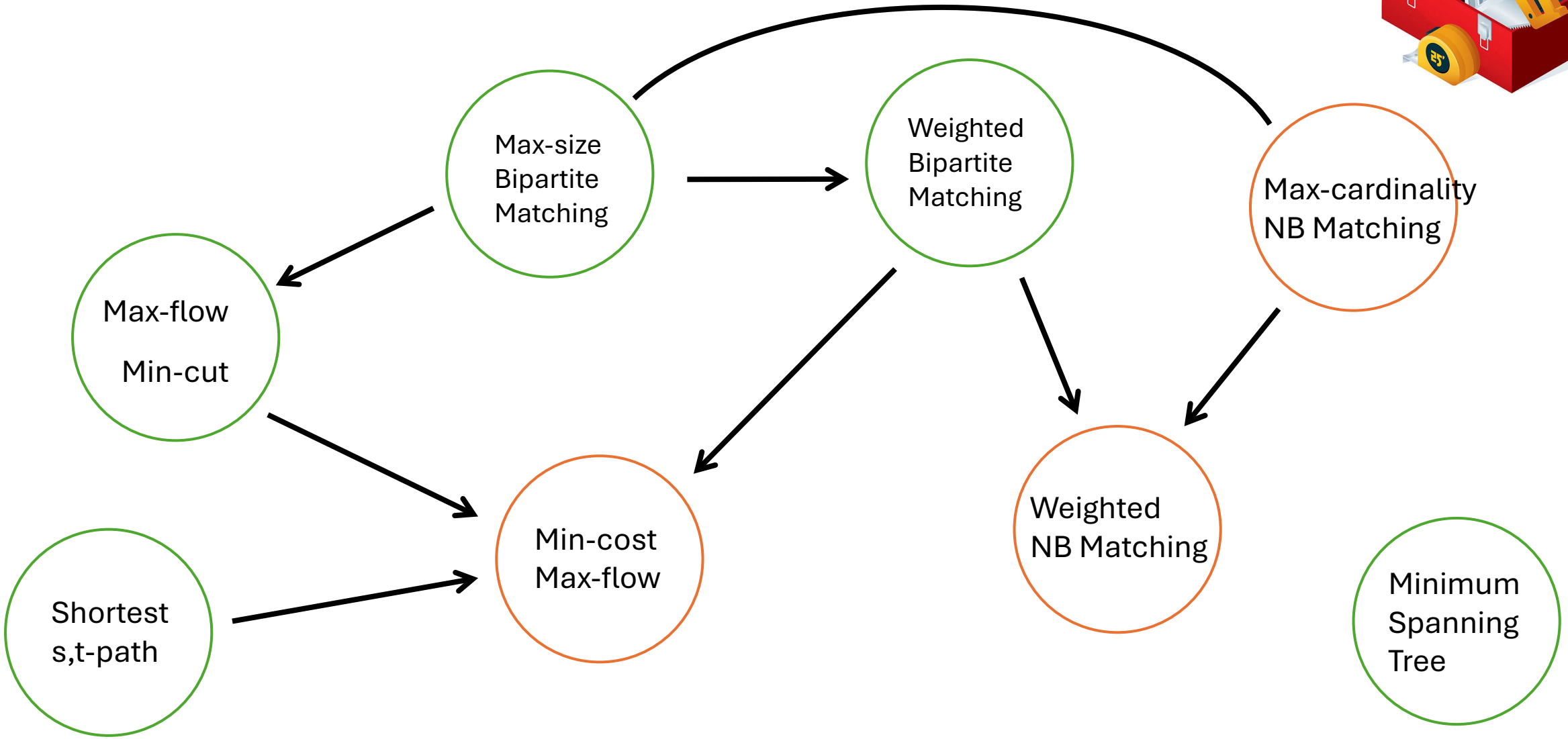
- Course evals

- Bit about my research

- Top 10 list: takeaways from the course

- Final Projects AMA

Fundamental Problems in P



Linear and Integer Programming

x_A = number of Aqua-Spas to produce

x_H = number of Hydro-Luxes to produce

Maximize: $350x_A + 300x_H$

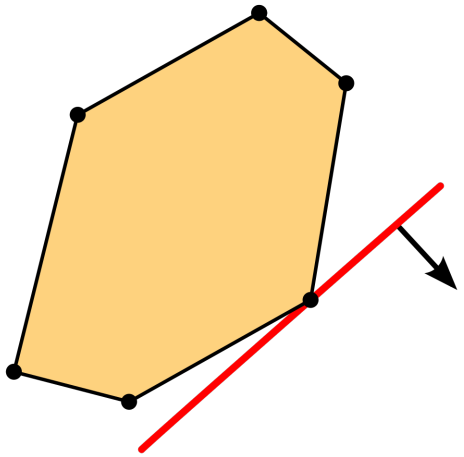
Subject to:

$x_A + x_H \leq 200$	(pumps)
$9x_A + 6x_H \leq 1566$	(labor)
$12x_A + 16x_H \leq 2880$	(tubing)
$x_A, x_H \geq 0$	(non-negativity)

Optimal value: $66,100

Linear Programming

- A very general problem in P. Already models many problems
- Used as a subroutine in modern algorithm design
  - Approximation Algorithms
  - Online Algorithms
- An expressive language for all optimization problems



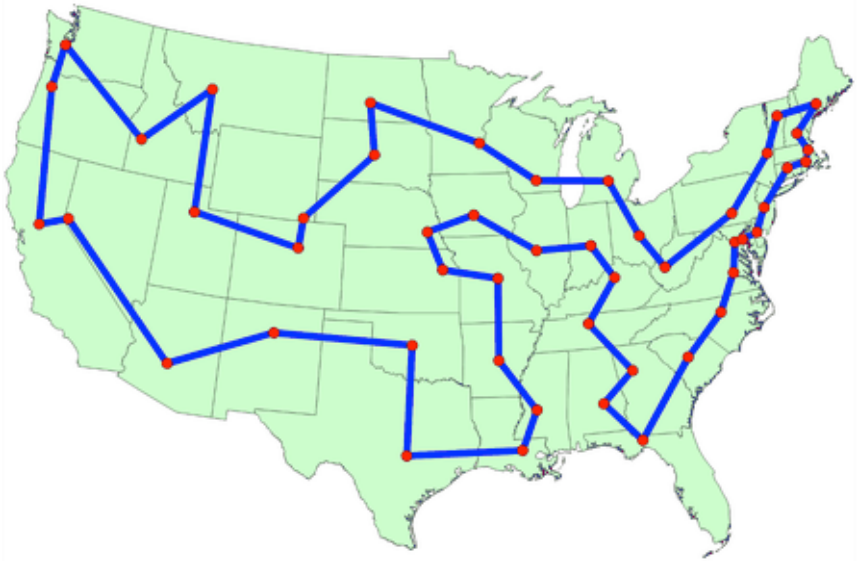
Duality

Dealing with NP-hard problems

- Restricting the class of instances under consideration (Vertex-Cover in bipartite graphs, Metric TSP)

- Exponential time algorithms
  - Parameterized Algorithms
  - Dynamic Programming

- Approximation algorithms
  - Scheduling
  - Clustering
  - Traveling Salesperson
  - Set Cover



Online Algorithms

We must make decisions online without full information.

Consider:

- Ski-rental, Online caching
- Matching users to rides
- Matching ad slots to advertisers

Maintain a solution which is competitive with the clairvoyant optimum.

Online Algorithms

We must make decisions online without full information.

Techniques:

- Better-late-than never
- Caching analysis
- Primal-dual analysis
- **Randomization is provably useful**

New Directions

- You are now capable of accessing and utilizing results at the forefront of algorithms
- Segue: your final project!
- Take on something yourself and share it with us